Faculty of Computers and Artificial Intelligence

CSx25: Digital Signal Processing
NCS224: Signals and Systems

Assignment no 04: Chapter 4

Note: You can check the exercises after the book Chapter.
In our assignment, we are using the first edition of "Signals and Systems: A MATLAB Integrated Approach" By Oktay Alkin.

Problems

4.2. Consider the pulse train shown in Fig. P.4.2.

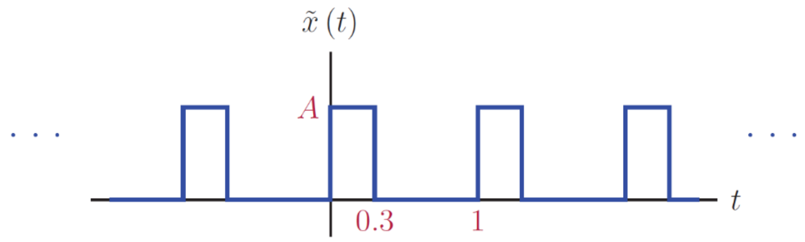

Figure P. 4.2

a. **Determine** the fundamental period T_0 and the fundamental frequency ω_0 for the signal.

b. **Find** an approximation to $\tilde{x}(t)$ in the form

$$\tilde{x}^{(1)}(t) \approx a_0 + a_1 \cos(\omega_0 t) + b_1 \sin(\omega_0 t)$$

Determine the optimum coefficients a_0, a_1 and b_1.

4.3. Consider again the pulse train shown in Fig. P.4.2.

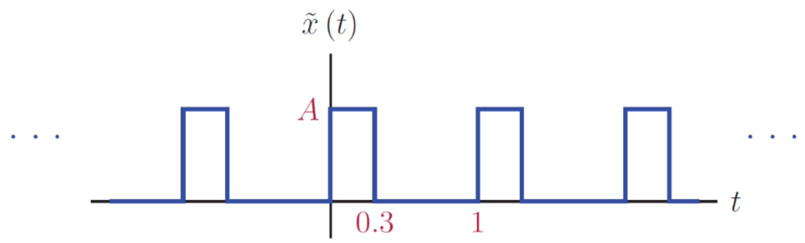

Figure P. 4.2

Find an approximation to $\tilde{x}(t)$ in the form

$$\tilde{x}^{(2)}(t) \approx a_0 + a_1 \cos(\omega_0 t) + b_1 \sin(\omega_0 t) + a_2 \cos(2\omega_0 t) + b_2 \sin(2\omega_0 t)$$

4.5. Consider the pulse train $\tilde{x}(t)$ shown in Fig. P.4.5.

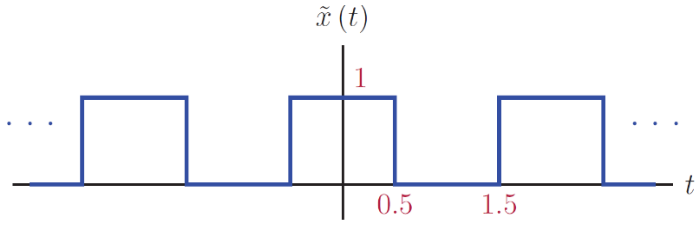

Figure P. 4.5

a. **Determine** the fundamental period T_0 and the fundamental frequency ω_0 for the signal.

b. **Determine** the coefficients of the approximation

$$\tilde{x}^{(2)}(t) = a_0 + a_1 \cos(\omega_0 t) + a_2 \cos(2\omega_0 t)$$

to the signal $\tilde{x}(t)$ that results in the minimum mean-squared error.

4.18. Find the Fourier transform of each of the pulse signals given below:

a. $x(t) = 3\,\Pi(t)$

c. $x(t) = 2\,\Pi\left(\frac{t}{4}\right)$

4.21. Refer to the signal shown in Fig. P.4.19.

$$\Pi(t-0.5) - \Pi(t-1.5)$$

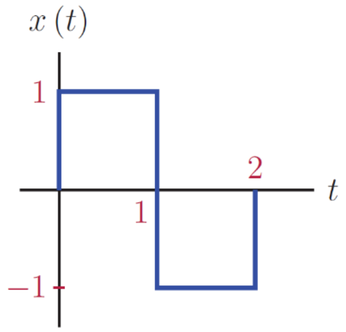

Figure P. 4.19

Find its Fourier transform by starting with the transform of the unit pulse and using **linearity** and **time shifting** properties.

4.24. Consider the following transform pair

$$e^{-a|t|} \overset{\mathcal{F}}{\longleftrightarrow} \frac{2a}{a^2+\omega^2}$$

Using this pair along with the **duality property**, **find** the Fourier transform of the signal

$$x(t) = \frac{2}{1+4t^2}$$

4.38. Determine and **sketch** the power spectral density of the following signals:

a. $x(t) = 3\cos(20\pi t)$

b. $x(t) = 2\cos(20\pi t) + 3\cos(30\pi t)$

c. $x(t) = 5\cos(200\pi t) + 5\cos(200\pi t)\cos(30\pi t)$

Examples

Example 4.1: A pulse-train signal $\tilde{x}(t)$ with a period of $T_0 = 3$ seconds is shown in Fig. 4.5. **Determine** the coefficients of the TFS representation of this signal.

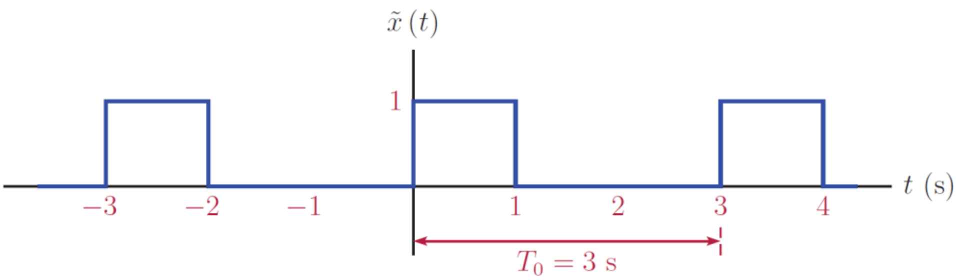

Figure 4.5 – The periodic pulse train used in Example 4.1.

Example 4.2: Approximate the periodic pulse train of Example 4.1 using

a. The first 4 harmonics.

b. The first 10 harmonics.

Example 4.3: Determine the TFS coefficients for the periodic pulse train shown in Fig. 4.7.

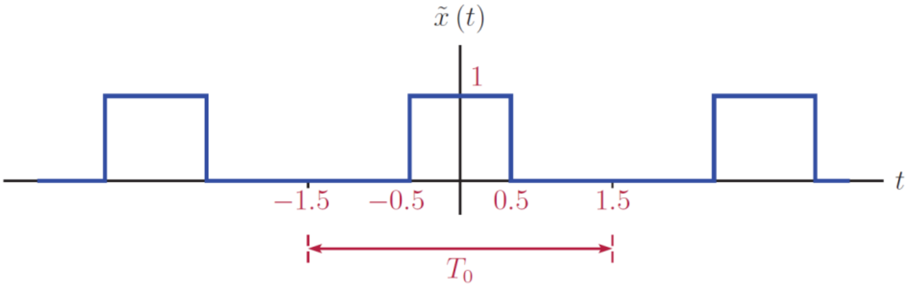

Figure 4.7 – The periodic pulse train used in Example 4.3.

Example 4.5: Determine the EFS coefficients of the signal $\tilde{x}(t)$ shown in Fig. 4.7.

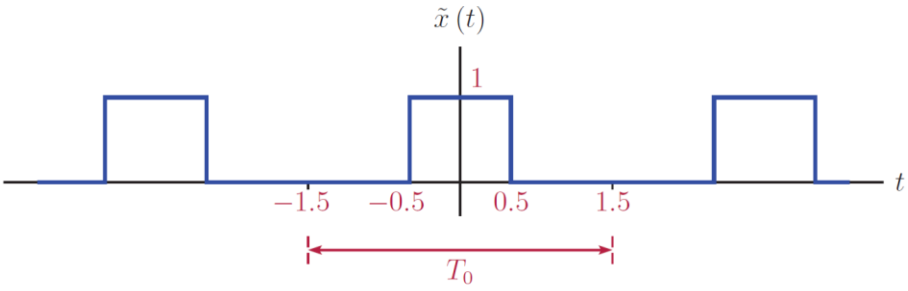

Figure 4.7 – The periodic pulse train used in Example 4.3.

Example 4.9: Determine the EFS coefficients and **graph** the line spectrum for the multi-tone signal shown in Fig. 4.19.

$$\tilde{x}(t) = \cos\left(2\pi\,[10 f_0]\,t\right) + 0.8\,\cos\left(2\pi f_0 t\right)\,\cos\left(2\pi\,[10 f_0]\,t\right)$$

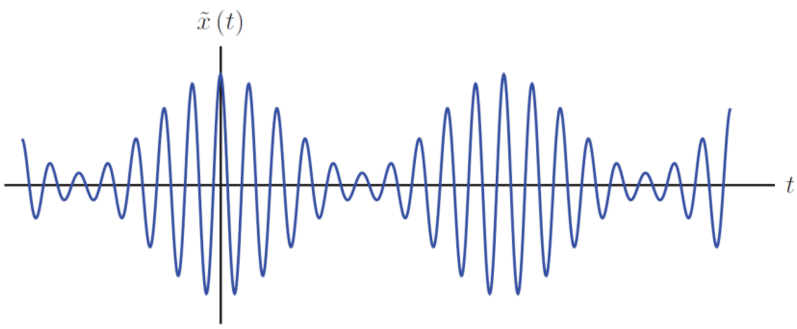

Figure 4.19 – Multi-tone signal of Example 4.9.

Example 4.12: Using the forward Fourier transform integral, **find** the Fourier transform of the isolated rectangular pulse signal shown in Fig. 4.35.

$$x(t) = A\,\Pi\left(\frac{t}{\tau}\right)$$

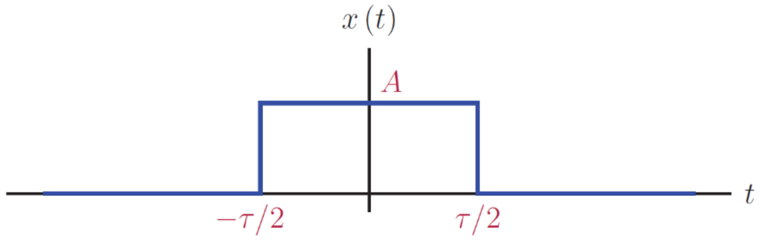

Figure 4.35

Example 4.15: Determine the Fourier transform of the right-sided exponential signal

$$x(t) = e^{-at}\,u(t)$$

with $a > 0$ as shown in Fig. 4.43.

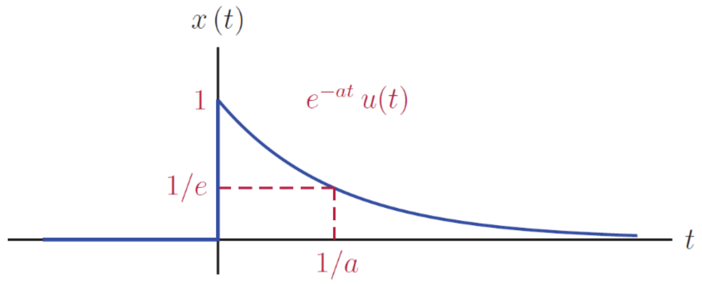

Figure 4.43 – Right-sided exponential signal.

Example 4.16: Determine the Fourier transform of the two-sided exponential signal given by

$$x(t) = e^{-a|t|}$$

where a is any non-negative real-valued constant. The signal $x(t)$ is shown in Fig. 4.46.

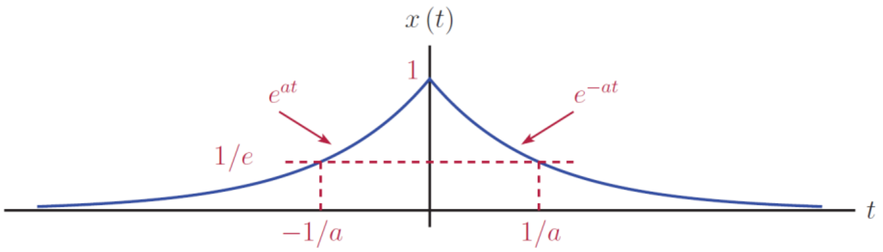

Figure 4.46 – Two-sided exponential signal $x(t)$.

Example 4.39: Find the power spectral density of the signal $\tilde{x}(t) = 5\cos(200\pi t)$.